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ROGUE WAVE AND SOLITON SOLUTION FOR THE NONLINEAR INTEGRABLE SPIN MODEL

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Abstract. In this paper, we explore the properties of soliton and rogue wave solutions of a nonlinear integrable equation using the Darboux transformation method, and discuss the possible practical applications of this research in various fields. These equations are important mathematical models for describing various wave phenomena in different physical systems, such as sound waves, light waves, and water waves. The Darboux method us allows to construct various interesting solutions like soliton and rogue wave solutions for nonlinear integrable equations. By studying the properties and behavior of soliton and rogue wave solutions, scientists could potentially develop new technologies and safety measures, especially in the fields of optical communication and marine engineering.

Key words: soliton solutions, nonlinear integrable equations, Darboux method, rogue waves
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СЫЗЫҚТЫ ЕМЕС ИНТЕГРАЛДАНАТЫН СПИНДІК МОДЕЛІ ҮШІН ҚИРАТУШЫ ТОЛҚЫН ЖӘНЕ СОЛИТОНДЫ ШЕШІМДЕР

Аннотация. Бұл мақалада біз Дарбу әдісін қолдана отырып, сызықты емес интегралданатын теңдеулердің солитондық және қиратушы толқын шешімдерінің қасиеттерін зерттейміз және осы зерттеудің әртүрлі салалардағы әлеуетті практикалық қолданылуын талқылаймыз. Бұл теңдеулер дыбыс толқындары мен жарық толқындары, теңіз толқындары сияқты әртүрлі физикалық жүйелердегі толқындық құбылыстарды сипаттайтын маңызды математикалық модельдер болып табылады. Дарбу әдісі сызықты емес интегралданатын теңдеулердің дәл солитонды және қиратушы толқын шешімдерін құруға мүмкіндік береді. Солитон және қиратушы толқын қасиеттерін зерттеу арқылы ғалымдар жаңа технологиялар мен қауіпсіздік шараларын, әсіресе оптикалық байланыс пен теңіз техникасы саласында жаңа ашу жасай алады.

Түйін сөздер: солитон шешімдері, сызықты емес интегралданатын теңдеулер, Дарбу әдісі, қиратушы толқын

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РАЗРУШИТЕЛЬНЫЕ ВОЛНЫ И СОЛИТОННОЕ РЕШЕНИЕ НЕЛИНЕЙНОЙ ИНТЕГРИРУЕМОЙ СПИНОВОЙ МОДЕЛИ

Аннотация. В этой статье мы исследуем свойства солитонных и разрушительных волновых решений нелинейных интегрируемых уравнений с использованием метода Дарбу и обсуждаем потенциальные практические применения данного исследования в различных областях. Эти уравнения являются важными математическими моделями для описания различных явлений волн в разных физических системах, таких как звуковые волны и световые волны, волны воды. Метод Дарбу позволяет строить точные солитонные и разрушительные решения нелинейных интегрируемых уравнений. Изучая свойства и поведение солитонных и разрушительных волновых решений, ученые могут потенциально разрабатывать новые технологии и меры безопасности, особенно в области оптической связи и морского инжиниринга.

Ключевые слова: солитонные решения, нелинейные интегрируемые уравнения, метод Дарбу, разрушительные волны

Introduction

Nonlinear wave behavior in physical systems has been a topic of research interest for many years, with the goal of gaining a better understanding of the underlying physics of these systems. We will investigate a rogue wave and soliton solution for our model, which is a nonlinear integrable equation.

A soliton solution is a type of nonlinear wave that maintains its shape and speed as it propagates through a medium. On the other side, rogue waves characterized by their extreme height and steepness. Mostly rogue waves are several times larger than the surrounding waves. They can suddenly appear and destroy everything. Because of it rogue waves claimed to as monster waves. In this paper, we will investigate the K-IAE equation (see notations in the work of (Nugmanova et al., 2022) and focus on the soliton and rogue wave solutions of the K-IAE using the Darboux method. This method is a powerful mathematical tool that can be used to construct explicit solutions for nonlinear integrable equations, making it an important tool for analyzing the properties of soliton and rogue wave solutions.

The relevance of solving these problems lies in their ability to accurately describe various wave phenomena in different physical systems. Understanding the behavior of these waves is important for gaining insights into the underlying physics of these systems and potentially developing new technologies and safety measures. They have applications in various fields such as hydrodynamics, electrodynamics, optical communication. For example, solitons have important applications in optical communication, while rogue waves are a major safety concern. These solutions are a valuable tool for analyzing the properties of phenomena. Overall, finding soliton solution and rogue wave solutions for our model is an important part of understanding wave phenomena and could have significant practical applications in various fields.

The aim of investigating the rogue wave and soliton solution is to find a new solution with a certain method and to improve our knowledge of the behavior of nonlinear waves in physical systems.

In the first section of our paper, we extensively investigate the model and examine its Lax representations, as well as discussing the origin of this model and thoroughly studying the properties of the S-matrix. In the second section, we explain in detail the method used for new solutions, which is a highly popular method in the field of solitons and widely used by soliton specialists. In this section, we add new values to old solutions to make them consistent with old data and use the L-matrix to obtain new values for the scalar potential u'. In the third section, we use the new values for S' and u' to find soliton and rogue wave solutions for our model. We choose an appropriate solution to the linear spectral problem to obtain new solutions for the required parameters We find soliton solutions in the form of (3.11)–(3.12) and use seed solutions to obtain parameters for the rogue wave solutions, which are shown in equations (3.22)–(3.23), (3.30)–(3.31). In conclusion, we create visualizations of these waves using the mathematical software Maple for soliton and rogue wave solutions to obtain a more detailed understanding of the properties of our model.

Methods and materials

Section 1. Model and its Lax representation. The Heisenberg ferromagnet equation is a model in statistical mechanics that describes the dynamics of a collection of spins in a ferromagnetic material. On the other hand, the nonlinear Schrödinger equation (NLSE) is a well-known equation in nonlinear optics and other areas of physics. Remarkably, it turns out that the Heisenberg ferromagnet equation can be mapped onto the nonlinear Schrödinger equation via a gauge transformation. This means that the two equations are equivalent in some sense, even though they describe very different physical systems. (Zakharov, 1979; Hoffmann, 1999; Ding, 2000; Myrzakulov et al., 1998; Makhankov et al., 1998) proved that the two equations are equivalent found connection between their solutions and integral of motions.

This equivalence is an example of the deep connections that exist between seemingly disparate areas of physics, and highlights the power of mathematical techniques in unifying our understanding of the natural world. Gauge equivalence of these equation allows us to obtain infinite number of conservation laws and solutions for both systems which also would be equivalent. In general, this equivalence is a powerful tool that help us to apply techniques from one field to the other, and has led to many important insights in both statistical mechanics and nonlinear optics.

One of the remarkable example of such nonlinear model is nonlinear Schrödinger equation. There are exact solutions of the Schrödinger equation that transform and precisely determine and reveal new equations using various solution methods, including the Darboux method. Using these methods, scientists have found new equations analogous to the Heisenberg equation. From papers (Myrzakulova et al., 2022; Myrzakulov et al., 2022) we know that is the anisotropic ZE-equation, has the following form:

$$\left(1+2\beta(c\beta+d)\right)\mathbf{S}_t-\mathbf{S}\Lambda\mathbf{S}_{xt}-u\mathbf{S}_x-2\beta(c\beta+d)\mathbf{S}_t+4c\omega\mathbf{S}_x=0, \tag{1.1}$$

$$u_x+\frac{1}{2}(\mathbf{S}_x^2)_t=0, \tag{1.2}$$

$$\omega_x+\frac{1}{4(2\beta c+d)^2}(\mathbf{S}_x^2)_t=0, \tag{1.3}$$

Assuming that c=0, the system of equations (1.1) - (1.3) was rewritten in the form

$$(1+2\beta d)\mathbf{S}_t-\mathbf{S}\Lambda\mathbf{S}_{xt}-u\mathbf{S}_x=0,, \tag{1.4}$$

$$u_x+\frac{1}{2}(\mathbf{S}_x^2)_t=0. \tag{1.5}$$

In order to obtain a new equation, we set B=0 and obtain the spin equation, or K-IA equation in vector form

$$\mathbf{S}_t-\ \mathbf{S}\Lambda\mathbf{S}_{xt}-u\mathbf{S}_x=0, \tag{1.6}$$

$$u_x+\frac{1}{2}(\mathbf{S}_x^2)_t=0, \tag{1.7}$$

where $\mathbf{S}\ =\ (S_1,S_2,S_3)$ is a spin vector with length $S^2\ =1$.

We know that the KE can have different variants, which are gauge equivalent to each other. In our case, we will use the Darboux transformation to find solutions for the K–IA equation.

Equation (1.6)–(1.7) is integrable and admits a Lax representation. A Lax pair is a mathematical tool used in the study of nonlinear integrable equations. It consists of a pair of linear differential equations and a compatibility condition between them. A Lax pair is a way to transform a difficult nonlinear equation into two simpler linear equations, which can be easier to solve. The compatibility condition ensures that the solutions of the linear equations are also solutions of the original nonlinear equation. By proving the existence of these pairs, we can establish that the given equation is integrable and find its solutions. The Lax pair for this equation is written as follows in the research of (Nugmanova et al., 2022):

$$\psi_x = U\psi, \quad (1.8)$$
$$\psi_t = V\psi, \quad (1.9)$$

where

$$U = -i\lambda S, \quad (1.10)$$
$$V = \frac{2\lambda}{1-2\lambda} Z, \quad (1.11)$$

where

$$Z = 0.25([S, S_t] + 2iuS), \quad (1.12)$$

u is a scalar potential and spin vector presented in matrix form

$$S = \begin{pmatrix} S_3 & S^- \\ S^+ & -S_3 \end{pmatrix}, \quad (1.13)$$
$$S^{\pm} = s_1 \pm is_2. \quad (1.14)$$

Section 2. Darboux transformation methods. A Darboux transformation is a popular method for constructing new solutions arising from known solutions. It is a systematic way of generating a family of solutions by introducing an auxiliary function that satisfies a linear equation, called the spectral problem. This method was studied in the scientific papers of (Chen Chi et al., 2009; Nian- Bing et al., 1989; Schief et al., 2002; Yersultanova et al.., 2016). In this article we will use the following technique: let us enter new function

$$\psi' = L\psi, \quad (2.1)$$

where L is a Darboux transforming matrix

$$L = \lambda N - I. \quad (2.2)$$

ψ' also satisfies the identical Lax representation in the form:

$$\psi'_x = U'\psi', \quad (2.3)$$
$$\psi'_t = V'\psi', \quad (2.4)$$

L matrix we consider from an article of Myrzakulov R. et al. (2015). It is easy to prove that the matrix L satisfies the equations

$$L_x + LU = U'L, \quad (2.5)$$
$$L_t + LV = V'L, \quad (2.6)$$

and we get

$$\lambda N_t + \frac{0.5}{1-2\lambda} N[\boldsymbol{S}, \boldsymbol{S}_t] + \frac{i}{1-2\lambda} uN\boldsymbol{S} - \frac{0.5}{\lambda(1-2\lambda)}[\boldsymbol{S}, \boldsymbol{S}_t] -$$
$$-\frac{iuS}{\lambda(1-2\lambda)} - \frac{0.5}{1-2\lambda}[\boldsymbol{S}', \boldsymbol{S}'_t]N - \frac{iu'\boldsymbol{S}'N}{1-2\lambda} + \frac{0.5}{\lambda(1-2\lambda)}[\boldsymbol{S}', \boldsymbol{S}'_t] + \frac{iu'\boldsymbol{S}'}{\lambda(1-2\lambda)} = 0. \quad (2.7)$$

Now collecting λ by the same powers we obtain

$$u' = \frac{((N[S, S_t] - [S', S'_t]N) + NuS)}{2i} S'^{-1} N^{-1}, \quad (2.8)$$

In the same way, the new solution S'

$$S' = NSN^{-1}. \quad (2.9)$$

So finally, we obtained 1-n fold Darboux transformation for considered spin system in form (2.9). Or we can rewrite it for spin matrix components as following

$$S^{+'} = S^+ + i\left(\frac{(\lambda_1^{-1} - \lambda_2^{-1})\psi_1^*\psi_2}{\Delta}\right)_x, \quad (2.10)$$
$$S^{-'} = S^- - i\left(\frac{(\lambda_1^{-1} - \lambda_2^{-1})\psi_1\psi_1^*}{\Delta}\right)_x, \quad (2.11)$$
$$S'_3 = S_3 - i\left(\frac{\lambda_1^{-1}|\psi_1|^2 + \lambda_2^{-1}|\psi_2|^2}{\Delta}\right)_x, \quad (2.12)$$

here $\Delta = |\psi_1| + |\psi_2|$. Now we are ready to construct various types of soliton and solitonlike solutions using DT (2.8)–(2.12).

Section 3. Soliton and rogue wave solutions. In order to find exact solution we choose a solution of linear spectral problem as $\psi = (\psi_1, \psi_2)^T$, where

$$\psi_i = e^{i\theta_i}. \quad (3.1)$$

On purpose to find a new soliton solution for K-IA equation, we will consider the next expression of ψ_1 and ψ_2:

$$\psi_1 = e^{i\theta_1}, \quad (3.2)$$
$$\psi_2 = e^{i\theta_2}, \quad (3.3)$$

here, θ is a parameter that determine the shape and behavior of the soliton and defines as

$$\theta_1 = mx - kt + \Delta, \quad (3.4)$$
$$\theta_2 = -(mx - kt + \Delta), \quad (3.5)$$

where *m, k,* and Δ are some constants. From the work of Drazin P.G. and Johnson R. S we know a seed solution as $S_3 = 0$ and

$$S^+ = f sech\big(c(mx - kt + \phi)\big)e^{(i(mx-kt-\Delta))}, \quad (3.6)$$

$$S^{-} = fsech\big(c(mx - kt + \phi)\big)e^{-(i(mx-kt-\Delta))}, \tag{3.7}$$

here f, ϕ, and γ are constant parameters.

In soliton theory, seed solution is a fundamental building block for constructing other solutions of the same type. Solitons are nonlinear waves with constant shape propagating. A seed solution is typically a simple soliton solution that can be used as a starting point for constructing more complex solutions. For example, by applying certain transformations to a seed solution, such as translations and reflections, one can generate new solutions with different shapes and speeds. The importance of seed solutions in soliton theory lies in their ability to generate a wide range of other solutions through these transformations. This allows researchers to study the properties of solitons in a systematic way and gain insights into their behavior and interactions. Seed solutions are an important tool in soliton theory that enable researchers to explore the rich and fascinating world of nonlinear waves. Substituting our seed solution and ψ we get

$$(\psi_1^*\psi_2)_x = -2ie^{-i(\theta_1-\theta_2)} \tag{3.8}$$

$$(\psi_1\psi_2^*)_x = 2ie^{-i(\theta_1+\theta_2)} \tag{3.9}$$

and

$$\Delta = \psi_1^*\psi_1 + \psi_2^*\psi_2 \tag{3.10}$$

After some calculations we get new soliton solutions for the K-IA equation

$$S^{+'} = fsech\big(c(mx - kt + \phi)\big)e^{(i(mx-kt-\Delta))} + (\lambda_1^{-1} - \lambda_2^{-1})me^{-2i(mx-ct+\Delta)}, \tag{3.11}$$

$$S^{-'} = fsech\big(c(mx - kt + \phi)\big)e^{-(i(mx-kt-\Delta))} + (\lambda_1^{-1} - \lambda_2^{-1})me^{2i(mx-ct+\Delta)}. \tag{3.12}$$

For the scalar u' of our model as equation (2.8) we get figure (1g) and visual representation of obtained solutions (3.11) – (3.12) we present on figures (1a)–(1f)

First rogue wave solution. A rogue wave in a soliton context refers to an unusually large and rare wave that appears suddenly and disappears quickly. It is a type of ocean wave that is much taller than the surrounding waves and can pose a significant danger to ships and structures in the ocean. Rogue waves solution and soliton solution were studied in the scientific articles of (Guo et al., 2021; Sagidullayeva et al., 2022).

As a soliton, a rogue wave occurs when several solitons interfere with each other, leading to a wave with much higher amplitude than the individual solitons. This interference can happen in a variety of ways, such as the merging of two solitons, the interaction of solitons with random waves, or the reflection of solitons from a boundary. In electrodynamics, a rogue wave refers to an extremely intense and localized electromagnetic wave that occurs unexpectedly in a medium. This wave is also known as an optical rogue wave, as it is often observed in optical fibers and other waveguides.

Rogue wave solutions demands plane wave solutions as a seed solution

$$S^{+} = 5n\sec(n(x - 7nt))^2\, e^{(x-7nt)}, \tag{3.13}$$

$$S^{-} = 5n\sec(n(x - 7nt))^2\, e^{-(x-7nt)}, \tag{3.14}$$

$$S_3 = 0.$$

To find a new rogue wave solution for the K-IA equation, we will examine the following expressions for ψ_3 and ψ_4 as:

$$\psi_3 = e^{i\theta_3}, \tag{3.15}$$

$$\psi_4 = e^{i\theta_4}, \tag{3.16}$$

for this case the parameter θ is specified as:

$$\theta_3 = x - 7nt, \tag{3.17}$$

$$\theta_4 = -(x - 7nt), \tag{3.18}$$

where n is a constant, which determines the scale of the wave. By using equations (3.15) – (3.16) and (3.17) – (3.18), we get the next expression:

$$(\psi_3^*\psi_4)_x = -2ie^{-2i(x-7nt)} \tag{3.19}$$

$$(\psi_3\psi_4^*)_x = 2ie^{2i(x-7nt)} \tag{3.20}$$

and the same result was obtained for Δ in the form:

$$\Delta = \psi_3^*\psi_3 + \psi_4^*\psi_4 \tag{3.21}$$

And we get new rogue wave solutions for the K-1A equation in the form:

$$S^{+'} = 5n\sec\big(n(x - 7nt)\big)^2\, e^{(x-7nt)} + \Big(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\Big)e^{-2i(x-7nt)}, \tag{3.22}$$

$$S^{-'} = 5n\sec(n(x - 7nt))^2\, e^{-(x-7nt)} + \Big(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\Big)e^{2i(x-7nt)}, \tag{3.23}$$

Figures (2a) displays a graphical representation of equation (2.8), while figures (2b) - (2r) depict graphical representations of equations (3.22) - (3.23).

Second rogue wave solution. In order to obtain second rogue wave solutions, we will use seed solutions as:

$$S^{+} = \frac{g(j + lg^2 + rg^4)}{1 + qg^2} \tag{3.24}$$

$$S^{-} = \frac{g(j - lg^2 + rg^4)}{1 + qg^2}, \tag{3.25}$$

$$S_3 = 0.$$

where the parameters l, r, and j are considered to be constants and g regarded as:

$$g = e^{-\frac{(x^2+t^2)}{2}}$$

For this case, we will take ψ_5, ψ_6 and θ_5, θ_6 to be:

$$\psi_5 = e^{i\theta_5}, \quad (3.26)$$

$$\psi_6 = e^{i\theta_6}, \quad (3.27)$$

$$\theta_5 = -\frac{(x^2+t^2)}{2}, \quad (3.28)$$

$$\theta_6 = \frac{x^2+t^2}{2}, \quad (3.29)$$

As a result, we obtained secondary rogue wave solutions for our model in the following form:

$$S^{+'} = \frac{g(j+lg^2+rg^4)}{1+qg^2} + (\frac{1}{\lambda_1} - \frac{1}{\lambda_2})xe^{i(x^2+t^2)}, \quad (3.30)$$

$$S^{-'} = \frac{g(j+lg^2+rg^4)}{1+qg^2} - (\frac{1}{\lambda_1} - \frac{1}{\lambda_2})xe^{-i(x^2+t^2)}, \quad (3.31)$$

Figures (3a) - (3d) depict graphical representations of equations (3.30)–(3.31), whereas figure (3e) displays a graphical

representation of equation (2.8).

Results

Outcomes for the soliton solutions:

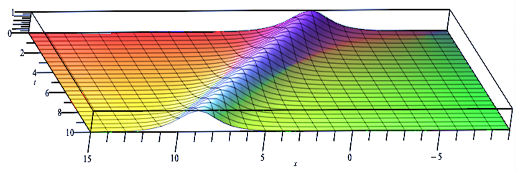

Figure 1(a). Graphic for the function $S^{+'}$ with parameters $f = 1, \gamma = 1, m = 1.5, \Delta = 1, \varphi = 1$.

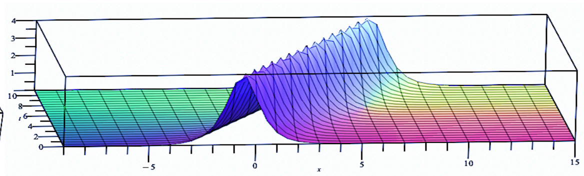

Figure 1(b). Graphic for the function $S^{+'}$ with parameters $f = 4, \gamma = 1, m = 1.5, \Delta = 1, \varphi = 1$.

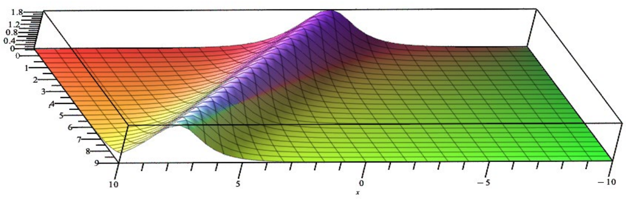

Figure 1(c). Graphic for the function $S^{-'}$ with parameters $f = 1.864353, \gamma = 1, m = 1, \Delta = 1, \varphi = 1$.

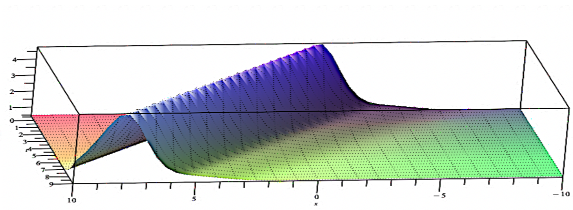

Figure 1(d). Graphic for the function $S^{-'}$ with parameters $f = 4, \gamma = 1, m = 1, \Delta = 1, \varphi = 1$.

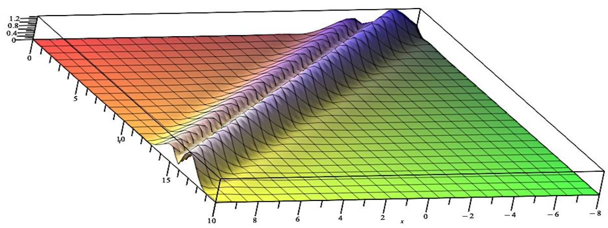

Figure 1(e). Graphic for the function $S_1{}'$ with parameters $f = 1.2, \gamma = 1, m = 1, \Delta = 1, \varphi = 1$, and where $S_1{}' = S^{+'} + S^{-'}$.

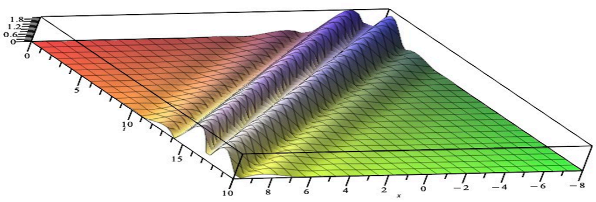

Figure 1(f). Graphic for the function $S_2{}'$ with parameters $f = 1.2, \gamma = 1, m = 1, \Delta = 1, \varphi = 7$, and here $S_2{}' = S^{+'} - S^{-'}$.

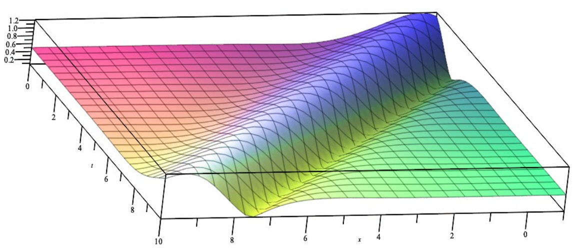

Figure 1(g). Graphic of the function u' with parameters $f = 1, \gamma = 1, m = 5, \Delta = 1, \varphi = 1$ for the soliton solution.

Outcomes for the first case of rogue wave solutions:

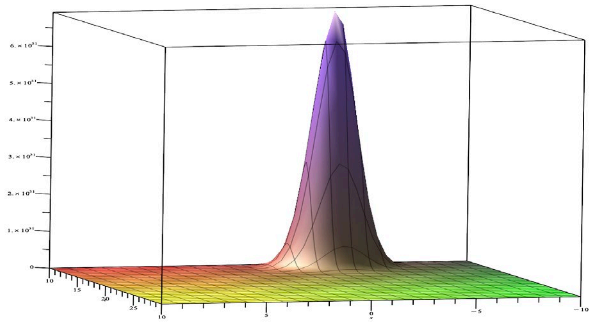

Figure 2(a). Graphic for the function $u'$with parameters n=1.

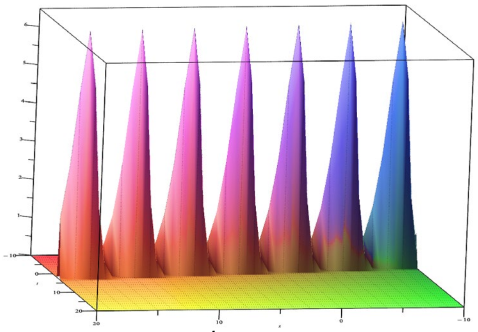

Figure 2(b). Graphic for the function $S^{+'}$where n = 1. The magnitude of the waves is determined by this n.

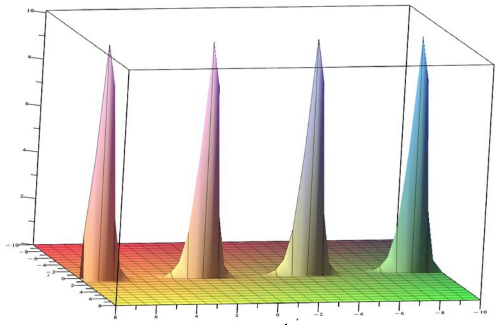

Figure 2(c). Graphic for the function $S^{+'}$where n = 2.

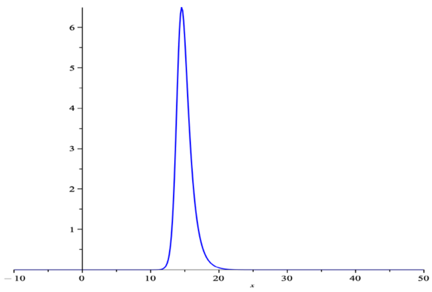

Figure 2(d). 2D Graphic for the function $S^{+'}$where x = 10 ... 20, t = 7.

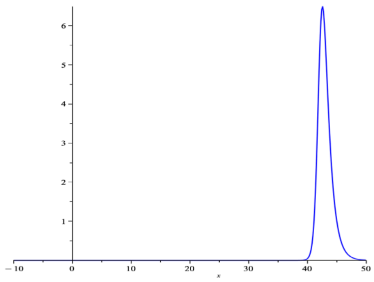

Figure 2(e). 2D Graphic for the function $S^{+'}$where n = 2. x = 40 … 50, t = 7.

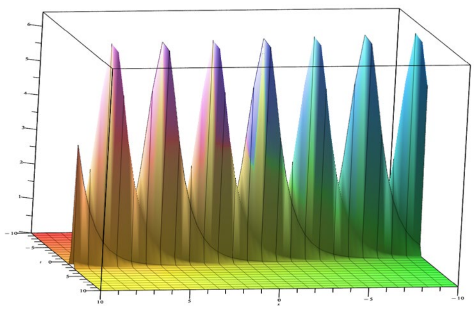

Figure 2(f). Graphic for the function $S^{-'}$where n = 1.

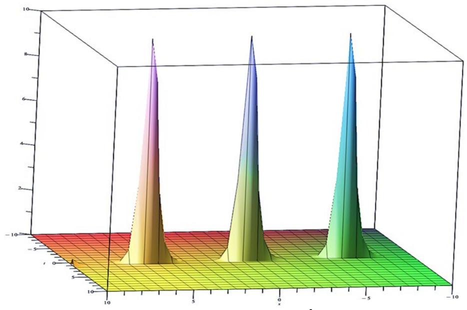

Figure 2(g). Graphic for the function $S^{-'}$where n = 2.

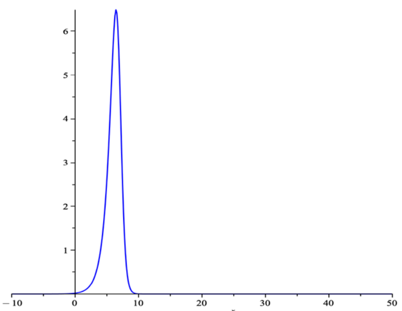

Figure 2(h). 2D Graphic for the function $S^{-'}$where x = -10 ...50, t = 2.

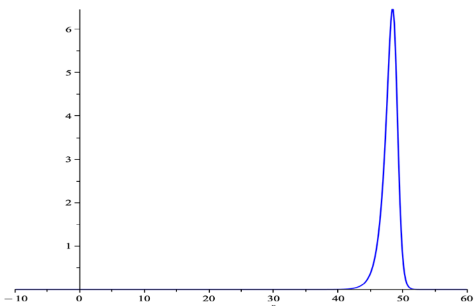

Figure 2(r). 2D Graphic for the function $S^{-'}$where n = 2. x = -10 ... 60, t = 7.

Outcomes for the second case of rogue wave solutions:

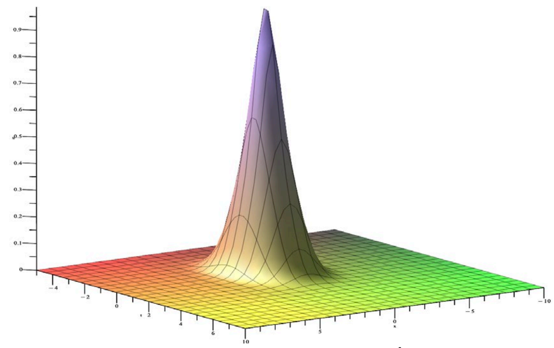

Figure 3(a). Graphic for the function $S^{+'}$ where j = 1, l = r =2, q = 4.

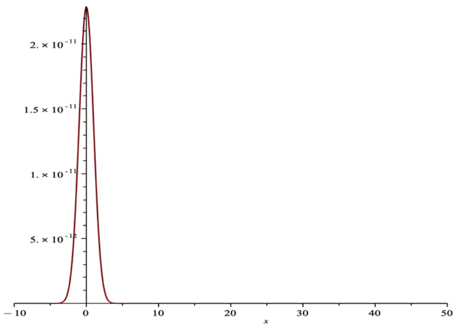

Figure 3(b). Graphic for the function $S^{+'}$. x = -10 … 50, t = 7 and l = r =2, q = 4, j = 1.

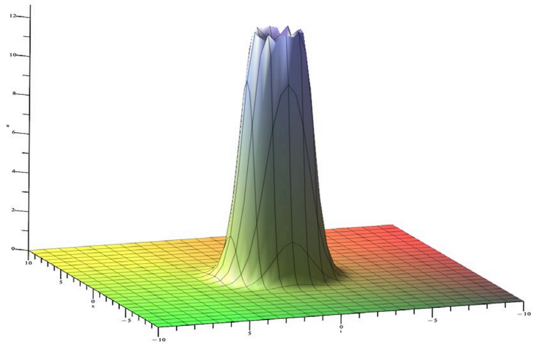

Figure 3(c). Graphic for the function $S^{-'}$ where j = 51, l = r =2, q = 4.

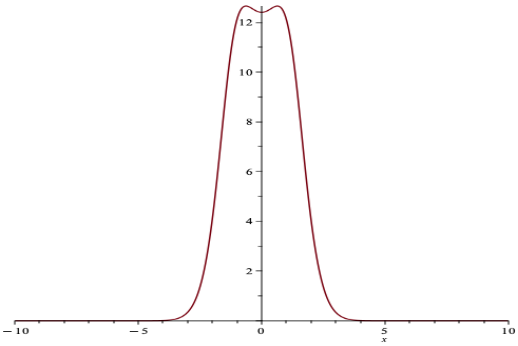

Figure 3(d). Graphic for the function $S^{-'}$. x = -10 … 10, t = 1 and l = r =2, q = 4, j = 51.

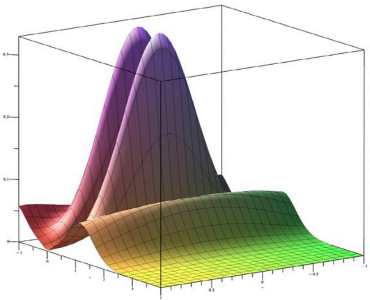

Figure 3(e1)

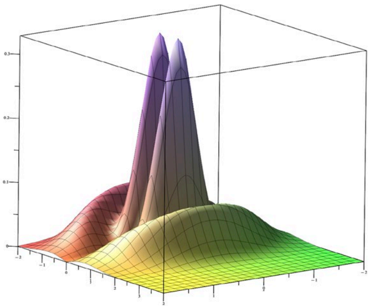

Figure 3(e2)

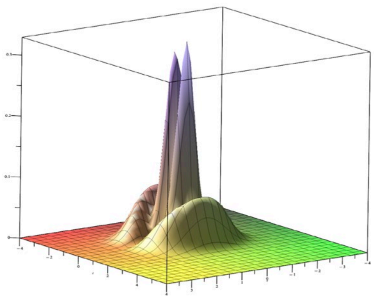

Figure 3(e3)

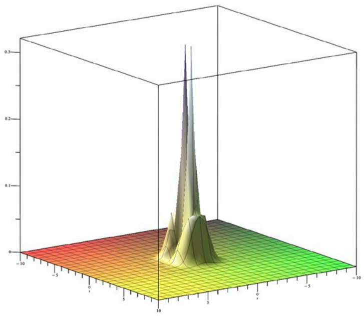

Figure 3(e4)

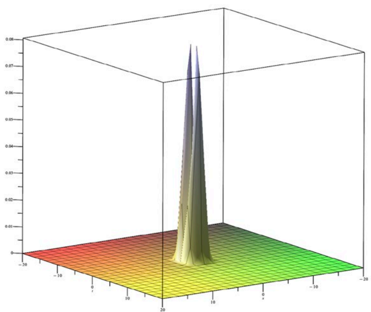

Figure 3(e5)

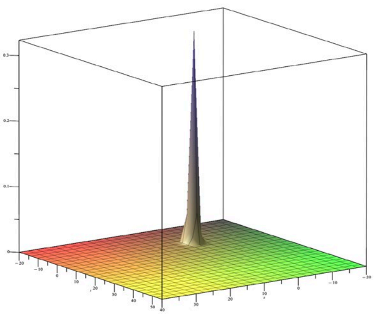

Figure 3(e6)

Figure 3(e). Graphic for the function u' for second case of rogue waves with values l = r = q = j = 1, x = -1 … 1, t = -1 … 1 for the figure 3(e1), x = -2 … 2, t = - 2 ... 2 for the figure 3(e2), x = -4 … 4, t = -4 … 4 for the figure 3(e3), x = -10 … 10, t = -10 … 10 for the figure 3(e4), x = -20 … 20, t = -20 … 20 for the figure 3(e5), and x = -20 … 40, t = -20 … 55 for the figure 3(e6).

Discussion

In our study, we found that the method used is suitable for our model. Using the Darboux transformation method, we obtained two different solutions, referred to as soliton and rogue waves. Additionally, with the help of the mathematical software Maple, we obtained representations of our model in soliton and rogue wave solutions. We understood that the method works and from our new values, graphs of rogue and soliton waves can be obtained.

Our analysis showed that the Darboux method is an effective tool for solving second-order differential equations with variable coefficients. This method allows the equation to be transformed into a canonical form, which can be easily solved using standard methods. However, we also identified some limitations of the Darboux method. This method is not suitable for all models, in particular, it can only be applied to linear second-order differential equations. Additionally, this method can be difficult to apply in some cases when the equation coefficients have a very complex form.

Conclusion

In this paper, we focused on examining the K-IA equation. We have confirmed the effectiveness of the transformation method used, leading to the discovery of new soliton and rogue wave solutions for the given equation. Additionally, a graph for the rogue wave solution was constructed using the mathematical software Maple, allowing for a clear demonstration of its properties and characteristics. As a result of our work, we have significantly expanded the understanding of this equation and its potential solutions, which may have important practical applications in various fields of science and technology.

We get soliton solutions for the spin operator S_2, defined as $S_2 = \frac{1}{2}(S^+ + S^-)$. Similarly, we considered the spin operator $S_1 = \frac{1}{2}(S^+ + S^-)$ and obtained the corresponding rogue wave solutions. Our results can be used for further exploration of the properties of systems with spin operators S_1 and S_2, as well as for the development of new methods for analyzing similar systems.

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