NEWS
OF THENATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X
Volume 1, Number 335 (2021), 6 – 13 **https://doi.org/10.32014/2021.2518-1726.1**

UDC 531.31; 519.21
MRNTI 27.29.17, 27.35.30

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ON INVERSE STOCHASTIC RECONSTRUCTION PROBLEM

Abstract. In this paper, general reconstruction problem in the class of second-order stochastic differential equations of the Ito type is considered for given properties of motion, when the control is included in the drift coefficient. And the form of control parameters is determined by the quasi-inversion method, which provides necessary and sufficient conditions for existence of a given integral manifold. Further, the solution of the Meshchersky’s stochastic problem is given, which is one of the inverse problems of dynamics and, according to the well-known Galiullin’s classification, refers to the restoration problem.

It is assumed that random perturbations belong to the class of processes with independent increments. To solve the posed problem an equation of perturbed motion is drawn up by the Ito rule of stochastic differentiation. And, further, the Erugin method in combination with the quasi-inversion method is used to construct: 1) a set of control vector functions and 2) a set of diffusion matrices that provide necessary and sufficient conditions for a given second-order differential equation of Ito type to have a given integral manifold.

The linear case of a stochastic problem with drift control is considered separately. In the linear setting, in contrast to the nonlinear formulation, the conditions of solvability in the presence of random perturbations from the class of processes with independent increments coincide with the conditions of solvability in a similar linear case in the presence of random perturbations from the class of independent Wiener processes. Also considered is the scalar case of the recovery problem with drift controls.

Key Words: Ito stochastic differential equation, reconstruction problem, Meshchersky's problem, integral manifold, quasi-inversion method.

Introductionю The theory of inverse problems of the dynamics of systems described by ordinary differential equations [1-6, etc.] goes back to the fundamental work of Erugin [7]. In 7, a set of ordinary differential equations is constructed that have a given integral curve. The inverse problems of constructing automatic control systems for program motion are studied in [8-11]. It should be noted that one of the general methods for solving inverse problems of dynamics in the class of ordinary differential equations, namely, the quasi-inversion method was proposed in [3].

Inverse problems of dynamics in the class of partial differential equations are studied in [12-14], and in the class of stochastic differential equations in [15-19].

1 Stochastic reconstruction problem

Let us consider the general reconstruction problem in the class of second-order Ito stochastic differential equations by the given properties of motion, when the control is included in the drift coefficient. And by the quasi-inversion method we define the form of control parameters that provides necessary and sufficient conditions for the existence of a given integral manifold. Further, the solution of Meshchersky's stochastic problem is given, which is one of the inverse problems of dynamics and it belongs to reconstruction problem according to the well-known Galiullin’s classification of [1].

1.1 Problem statement. Let us consider the second-order Ito stochastic differential equation

$$\ddot{x} = f(x,\dot{x},t) + D(x,\dot{x},t)u + \sigma(x,\dot{x},t)\dot{\xi}. \tag{1.1}$$

It is required to determine the vector-function $u = u(x,\dot{x},t) \in R^r$ included in the drift coefficient for the given integral manifold

$$\Lambda(t): \lambda(x,\dot{x},t) = 0, \quad \lambda = \lambda(x,\dot{x},t) \in C^{121}_{x\dot{x}t}, \quad \lambda \in R^m, \tag{1.2}$$

here $C^{121}_{x\dot{x}t}$ is set of $\gamma(x,\dot{x},t)$ functions, which are continuously differentiable with respect to x and t and twice continuously differentiable with respect to $\dot{x}$.

In other words, for the given f, D, σ and λ, the control u should be defined so that the set (1.2) is the integral set of equation (1.1).

It is assumed that the functions $f(x,\dot{x},t)$, $D(x,\dot{x},t)$, $\sigma(x,\dot{x},t)$, included in the above equation, have the smoothness necessary for further reasoning and satisfy the existence and uniqueness theorem up to stochastic equivalence of the solution $(x(t)^T, \dot{x}(t)^T)^T$ of (1.1) with the initial condition $(x(t_0)^T, \dot{x}(t_0)^T)^T = (x_0{}^T, \dot{x}_0{}^T)^T$, which is continuous strictly Markov process with probability 1 [20].

Here $\{\xi_1(t,\omega),\ldots,\xi_k(t,\omega)\}$ is a system of random processes with independent increments, which, following [19], can be represented as a sum $\xi = \xi_0 + \int c(y)P^0(t,dy)$ of processes Wiener process ξ_0 and Poisson process P^0. $P^0(t,dy)$ is the number of process P^0 jumps in the interval [0, t] that fall on the set dy; $c(y)$ is a vector function that maps space R^{2n} into the space R^k of values of process $\xi(t)$ for any t.

This problem is one of the inverse problems of dynamics, and in the absence of random perturbations $(\sigma \equiv 0)$ it has been sufficiently fully investigated in [1-6], and the case $\sigma \neq 0$ and $\{\xi_1(t,\omega),\ldots,\xi_k(t,\omega)\}$ is a system of independent Wiener processes, as a particular form of processes with independent increments, is considered in [21].

In this paper, the quasi-inversion method is used to solve the stochastic recovery problem [3, p. 12].

By Ito rule of stochastic differentiation [20, p.204] for solving the posed problem the equation of perturbed motion

$$\dot{\lambda} = \frac{\partial\lambda}{\partial t} + \frac{\partial\lambda}{\partial x}\dot{x} + \frac{\partial\lambda}{\partial\dot{x}}f + \frac{\partial\lambda}{\partial\dot{x}}Du + \frac{\partial\lambda}{\partial\dot{x}}\sigma\dot{\xi}_0 + S_1 + S_2 + S_3, \tag{1.4}$$

is compiled. Here $S_1 = \frac{1}{2}\frac{\partial^2\lambda}{\partial\dot{x}^2} : \sigma\sigma^T$; $S_2 = \int[\lambda(x,\dot{x}+\sigma c(y),t) - \lambda(x,\dot{x},t) - \frac{\partial\lambda}{\partial x}\sigma\dot{x}c(y)]dy$;

$S_3 = \int[\lambda(x,\dot{x}+\sigma c(y),t) - \lambda(x,\dot{x},t)]\dot{P}_0(t,dy)$. Following [20], $\frac{1}{2}\frac{\partial^2\lambda}{\partial\dot{x}^2} : D$ is a vector whose elements are the traces of the products of the matrices of the second derivatives of the corresponding elements $\lambda_\mu(x,\dot{x},t)$ of the vector $\lambda(x,\dot{x},t)$ with respect to components $\dot{x}$ by the matrix D

$$\frac{\partial^2\lambda}{\partial\dot{x}^2} : D = \begin{bmatrix} tr\left(\frac{\partial^2\lambda_1}{\partial\dot{x}^2}D\right) \\ \vdots \\ tr\left(\frac{\partial^2\lambda_m}{\partial\dot{x}^2}D\right) \end{bmatrix}, \quad D = \sigma\sigma^T.$$

We introduce arbitrary Erugin functions [1]: an m-dimensional vector function A and a $(m \times k)$-matrix B, with the properties $A(0,x,\dot{x},t) \equiv 0$, $B(0,x,\dot{x},t) \equiv 0$, such that

$$\dot{\lambda} = A(\lambda,x,\dot{x},t) + B(\lambda,x,\dot{x},t)\dot{\xi}, \tag{1.5}$$

takes place. Here ξ is the same process with independent increments included in (1.1) and represented as a sum of Wiener process and Poisson process [20]:

$\xi = \xi_0 + \int c(y)P^0(t,dy)$ or $\dot{\xi} = \dot{\xi}_0 + \int c(y)\dot{P}^0(t,dy)$.

Based on equations (1.4) and (1.5), we obtain the relations

$$\frac{\partial \lambda}{\partial \dot{x}} Du = A - \frac{\partial \lambda}{\partial t} - \frac{\partial \lambda}{\partial x}\dot{x} - \frac{\partial \lambda}{\partial \dot{x}} f - S_1 - S_2 - S_3, \tag{1.6}$$

$$\frac{\partial \lambda}{\partial \dot{x}} \sigma = B, \tag{1.7}$$

from which you need to determine the control u and the matrix σ. To solve the problem, you need the following lemma.

Lemma 1[3, p.12-13]. The set of all solutions of a linear system

$$H\vartheta = g, \quad H = (h_{\mu\nu}), \quad \vartheta = (\vartheta_\nu), \quad g = (g_\mu), \quad \mu = \overline{1,m}, \quad \nu = \overline{1,n}, \quad m \le n, \tag{1.8}$$

is determined by the expression

$$\vartheta = s[HC] + H^+ g. \tag{1.9}$$

Here H is matrix has rank m. s is arbitrary scalar, $[HC] = [h_1 \dots h_m \; c_{m+1} \dots c_{n-1}]$ is the cross product of vectors $h_\mu = (h_{\mu\nu})$ and $c_\rho = (c_{\rho\nu})$, $\rho = \overline{m+1, n-1}$; $H^+ = H^T\left(HH^T\right)^{-1}$, H^T is the matrix transposed to H.

Denoting, by formula (1.9) of Lemma 1 from relations (1.6), (1.7), we define the required vector - function and columns, matrices in the form:

Denoting $\widetilde{D} = \frac{\partial \lambda}{\partial \dot{x}} D$, by formula (1.9) from (1.6), (1.7) we define the required vector – function u and columns σ_i, $i = \overline{1,k}$ of σ in the form :

$$u = s_1\left[\widetilde{D}C\right] + \left(\widetilde{D}\right)^+ \left(A - \frac{\partial \lambda}{\partial t} - \frac{\partial \lambda}{\partial x}\dot{x} - \frac{\partial \lambda}{\partial \dot{x}} f - S_1 - S_2 - S_3 \right), \tag{1.10}$$

$$\sigma_i = s_2\left[\frac{\partial \lambda}{\partial \dot{x}} C\right] + \left(\frac{\partial \lambda}{\partial \dot{x}}\right)^+ B_i, \quad i = \overline{1,k}. \tag{1.11}$$

Therefore, the following theorem is true.

Theorem 1.1 A necessary and sufficient condition that second-order Ito differential equation (1.1) has a given integral manifold (1.2) is that the control function u has the form (1.10) and the columns σ_i of diffusion matrix σ have the form (1.11).

Remark 1.1 If $c(y) \equiv 0$, then $S_2 \equiv S_3 \equiv 0$ and a solution of this problem coincides with the solution of the reconstruction problem previously considered in [21] in the presence of random perturbations from the class of independent Wiener processes.

Remark 1.2 For $m = n$, formula (1.9) takes the form $\vartheta = H^{-1}g$, since in this case, the first term of the formula as a cross product of n vectors in $n-$dimensional space identically equals to zero $[HC] \equiv 0$, and the second term $H^+ g$ takes the form $H^{-1}g$, since for $m = n$, the rectangular matrix H becomes square matrix, and under the assumption $\det H \ne 0$ we have $H^+ = H^T\left(HH^T\right)^{-1} = H^T (H^T)^{-1} H^{-1} = H^{-1}$.

1.2 The linear case of a stochastic problem with drift control. Let a second-order Ito stochastic differential equation, linear in drift

$$\ddot{x} = E_1(t)x + E_2(t)\dot{x} + D(t)u + l_1(t) + T(t)\dot{\xi} \qquad (1.12)$$

be given. It is required the control vector-function $u = u(x, \dot{x}, t) \in R^r$ by given integral manifold

$$\Lambda(t) : \lambda \equiv G_1 x + G_2 \dot{x} + l_2(t) = 0. \qquad (1.13)$$

That is, by the given $(m \times n)$-matrices $G_1(t), G_2(t)$ of the m-dimensional function $l(t)$, also by the given $(n \times n)$-matrices $E_1(t), E_2(t)$, $(n \times r)$-matrix $D(t)$ and n-dimensional function $l_1(t)$, it is required to determine the vector function $u = u(x, \dot{x}, t) \in R^r$ and $(n \times k)$-matrix $T(t)$ so that for the constructed equation (1.12) the given properties (1.13) are an integral manifold.

In this problem the equation of perturbed motion (1.4) has the form

$$\dot{\lambda} = \dot{G}_1 x + \dot{G}_2 \dot{x} + \dot{l}_2(t) + \dot{G}_1 \dot{x} + G_2\left(E_1(t)x + E_2(t)\dot{x} + D(t)u + l_1(t)\right) + G_2 T \dot{\xi}_0. \qquad (1.14)$$

On the other hand, following Erugin's method with the help of an arbitrary vector-function $A = A_1(t)\lambda$ and a matrix-function B_1 with the property $B_1(0, x, \dot{x}, t) \equiv 0$, we have

$$\dot{\lambda} = A_1(t)\lambda + B_1(\lambda, x, \dot{x}, t)\dot{\xi}. \qquad (1.15)$$

Hence, relations (1.14) and (1.15) imply the equalities

$$\begin{cases} G_2 D u = \left[A_1 G_1 - \dot{G}_1 - G_2 E_1\right]x + \left[A_1 G_2 - G_2 - G_1 - G_2 E_2\right]\dot{x} + A_1 l_2 - G_2 l_1 - \dot{l}_2, \\ G_2 T = B_1. \end{cases} \qquad (1.16)$$

Further, from (1.16) using Lemma 1, we have

$$u = s_1\left[D_1 C\right] + \left(D_1\right)^+ g_1, \quad (1.17)$$

$$T_i = s_2\left[G_2 C\right] + \left(G_2\right)^+ B_i, \quad i = \overline{1, k}, \quad (1.18)$$

here $D_1 = G_2 D$, $g_1 = \left[A_1 G_1 - \dot{G}_1 - G_2 E_1\right]x + \left[A_1 G_2 - G_2 - G_1 - G_2 E_2\right]\dot{x} + A_1 l_2 - G_2 l_1 - \dot{l}_2$, T_i, B_i are i-th columns of matrices T and B respectively. s_1, s_2 are arbitrary scalar values. This proves the following theorem.

Theorem 1.2. A necessary and sufficient condition that second-order Ito stochastic differential equation (1.12) linear in drift, has a given linear integral manifold (1.2) is that the control parameter has the form (1.17) and the diffusion matrix has the form (1.18).

Remark 1.2. In the linear case, in contrast to the nonlinear one $S_1 \equiv S_2 \equiv S_3 \equiv 0$, the conditions of solvability in Theorem 1.2 in the presence of random perturbations from the class of processes with independent increments coincide with the conditions of solvability in a similar linear case in the presence of random perturbations from the class of independent Wiener processes [21].

1.3 Scalar case of the reconstruction problem with drift controls. *Let a second-order Ito stochastic differential equation*

$$\ddot{x} = f_2(x, \dot{x}, t) + \gamma_1(x, \dot{x}, t)u_1 + \gamma(x, \dot{x}, t)\dot{\xi}. \qquad (1.20)$$

be given. It is required to determine the scalar function $u_1 = u_1(x, \dot{x}, t)$ for a given integral manifold

$$\lambda_2(x, \dot{x}, t) = 0, \quad \lambda_2 \in R^1. \qquad (1.21)$$

In other words, for given f_2, γ, γ_1 and λ_2 define the control parameter u_1 in such a way that the set (1.21) be an integral set of equation (1.20).

According to the rule of stochastic differentiation, we compose the equation

$$\dot{\lambda}_2 = \frac{\partial \lambda_2}{\partial t} + \frac{\partial \lambda_2}{\partial x}\dot{x} + \frac{\partial \lambda_2}{\partial \dot{x}} f_2 + \frac{\partial \lambda_2}{\partial \dot{x}}\gamma_1 u_1 + \widetilde{S}_1 + \widetilde{S}_2 + \widetilde{S}_3 + \frac{\partial \lambda_2}{\partial \dot{x}}\gamma \dot{\xi}_0, \qquad (1.22)$$

of perturbed motion; here

$$\widetilde{S}_1 = \frac{1}{2}\frac{\partial^2 \lambda_2}{\partial \dot{x}^2}\gamma^2, \quad \tilde{S}_3 = \int [\lambda_2(x, \dot{x} + \gamma c(y), t) - \lambda_1(x, \dot{x}, t)] dP^0(t, dy),$$

$$\widetilde{S}_2 = \int [\lambda_2(x, \dot{x} + \gamma c(y), t) - \lambda_2(x, \dot{x}, t) - \frac{\partial \lambda_2}{\partial \dot{x}}\gamma c(y)] dy.$$

We introduce arbitrary Erugin's scalar functions $a = a(\lambda_2, x, \dot{x}, t)$ and $b = b(\lambda_2, x, \dot{x}, t)$, with the properties $a(0, x, \dot{x}, t) \equiv b(0, x, \dot{x}, t) \equiv 0$ and such that

$$\dot{\lambda}_2 = a(\lambda_2, x, \dot{x}, t) + b(\lambda_2, x, \dot{x}, t)\xi, \ (1.23)$$

In view of (5.3) and (5.4), we arrive at the relations

$$\frac{\partial \lambda_2}{\partial x}\gamma_1 u_1 = a - \frac{\partial \lambda_2}{\partial t} - \frac{\partial \lambda_2}{\partial x}\dot{x} - \frac{\partial \lambda_2}{\partial \dot{x}} f_2 - \widetilde{S}_1 - \widetilde{S}_2 - \widetilde{S}_3, \tag{1.24}$$

$$\frac{\partial \lambda_2}{\partial x}\gamma = b. \tag{1.25}$$

Then, by virtue of equalities (1.24) and (1.25), the control parameter u_1 and the diffusion coefficient are defined in the form

$$u_1 = \left(\frac{\partial \lambda_2}{\partial \dot{x}}\gamma_1\right)^{-1}\left[a - \frac{\partial \lambda_2}{\partial t} - \frac{\partial \lambda_2}{\partial x}\dot{x} - \frac{\partial \lambda_2}{\partial \dot{x}} f_2 - \frac{1}{2}\frac{\partial^2 \lambda_2}{\partial \dot{x}^2}\gamma^2 - \widetilde{S}_2 - \widetilde{S}_3\right], \tag{1.26}$$

$$\gamma = \left(\frac{\partial \lambda_2}{\partial \dot{x}}\right)^{-1} b. \tag{1.27}$$

Consequently, the following theorem holds.

Theorem 1.3 A necessary and sufficient condition that second-order scalar differential equation of Ito type (1.20) has a given integral manifold (1.21) is that the control parameter u_1 has the form (1.26) and diffusion coefficient has the form (1.27).

Thus, in Section 1, we obtained necessary and sufficient conditions for the solvability of the reconstruction problem with drift control in the presence of random perturbations from the class of processes with independent increments in general nonlinear case, linear case and scalar nonlinear case. The considered setting generalizes the reconstruction problem in the presence of random perturbations from the class of independent Wiener processes, previously studied in [21].

2 Meshchersky's stochastic problem

Problem statement. Find the law of change in the mass of a point, at which it describes a given trajectory under the action of given external forces [22, p.19].

Let us consider the problem of realizing the motion of a heavy point of variable mass $m(t)$ in a homogeneous gravity field, namely, vertical ascent according to the laws of change in the range y and height z

$$\Lambda(t): \begin{cases} \lambda_1(t) \equiv y - \varphi(t) = 0, \\ \lambda_2(t) \equiv z - \psi(t) = 0, \end{cases} \tag{2.1}$$

The equations of point motion [1, c.16-17] taking into account the action of random perturbing forces have the following form:

$$\begin{cases} m\ddot{y} = \dot{m}(\mu - 1)\dot{y} - mf(z, v)\frac{\dot{y}}{v} - \sigma_1(y, z, t)\dot{\xi}, \\ m\ddot{z} = \dot{m}(\eta - 1)\dot{z} - mf(z, v)\frac{z}{v} - mg - \sigma_2(y, z, t)\dot{\xi}, \end{cases} \tag{2.2}$$

here $f(z,v)$ is medium resistance per unit mass; $v=\sqrt{\dot{y}^2+\dot{z}^2}$ is point speed; $\mu=\mu(t)$, $v=v(t)$ are the ratio of the projections of the velocities of the changing mass and the mass of the point itself on the coordinate axis y, z.

It is required to restore the equations of motion (2.2) (that is, to determine the laws of variation of quantities μ, v and m) so that they admit a given particular motion (2.1).

The perturbed motion equations have the form

$$\begin{cases} \ddot{\lambda}_1 \equiv \ddot{y}-\ddot{\varphi}(t)=\dfrac{\dot{m}}{m}(\mu-1)\dot{y}-f(z,v)\dfrac{\dot{y}}{v}-\dfrac{\sigma_1}{m}\dot{\xi}-\ddot{\varphi}(t), \\ \ddot{\lambda}_2 \equiv \ddot{z}-\ddot{\psi}(t)=\dfrac{\dot{m}}{m}(\eta-1)\dot{z}-f(z,v)\dfrac{\dot{z}}{v}-g-\dfrac{\sigma_2}{m}\dot{\xi}-\ddot{\psi}(t). \end{cases} \tag{2.3}$$

Further, following Erugin's method [7], we introduce functions $A_1=A_1(\lambda_1,\dot{\lambda}_1,\lambda_2,\dot{\lambda}_2,y,z,t)$, $A_2=A_2(\lambda_1,\dot{\lambda}_1,\lambda_2,\dot{\lambda}_2,y,z,t)$, $B_1=B_1(\lambda_1,\dot{\lambda}_1,\lambda_2,\dot{\lambda}_2,y,z,t)$, $B_2=B_2(\lambda_1,\dot{\lambda}_1,\lambda_2,\dot{\lambda}_2,y,z,t)$,
With the properties $A_1(0,0,0,0,y,z,t)\equiv A_2(0,0,0,0,y,z,t)\equiv B_1(0,0,0,0,y,z,t)\equiv$
$\equiv B_2(0,0,0,0,y,z,t)\equiv 0$, and such that

$$\begin{cases} \ddot{\lambda}_1 = A_1 + B_1\dot{\xi}, \\ \ddot{\lambda}_2 = A_2 + B_2\dot{\xi}. \end{cases} \tag{2.4}$$

Comparison of the systems of equations (2.3) and (2.4) leads, if we exclude strictly vertical and strictly horizontal motions (i.e., $\dot{\varphi}$ and $\dot{\psi}$ are not identically zero), to relations that solve the posed Meshchersky stochastic problem

$$\begin{cases} \mu = 1+\dfrac{m}{\dot{m}}\left[\dfrac{A_1}{\dot{y}}+\dfrac{f}{v}+\dfrac{\ddot{\varphi}}{\dot{y}}\right], \\ \eta = 1+\dfrac{m}{\dot{m}}\left[\dfrac{A_2}{\dot{z}}+\dfrac{f}{v}+\dfrac{g}{\dot{z}}+\dfrac{\ddot{\psi}}{\dot{y}}\right], \\ \sigma_{1j} = mB_{1j}, \\ \sigma_{2j} = mB_{2j}. \end{cases} \tag{2.5}$$

In particular, for $\sigma_{ij}\equiv 0$ $(i,j=1,2)$ and $A_1\equiv A_2\equiv B_1\equiv B_2\equiv 0$ conditions (2.5) coincide with conditions in the class of second-order ordinary differential equations [1, p. 17].

This research has been funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP 08955847).

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КЕРІ СТОХАСТИКАЛЫҚ ҚАЛПЫНА КЕЛТІРУ ЕСЕБІ ТУРАЛЫ

Аннотация. Аталмыш жұмыста жалпы қалпына келтіру есебі қозғалыс қасиеттері бойынша басқару қирату коэффициентіне кіргенде екінші ретті Ито түріндегі берілген стохастикалық дифференциалдық теңдеулер класында қарастырылады және квазиқайтару әдісімен интегралдық көпбейненің қажетті әрі жеткілікті шарттарын қамтамасыз ететін басқарушы параметрлер түрі анықталады. Төменде Мещерскийдің стохастикалық есебінің шешімі келтірілген, ол динамиканың кері есептерінің бірі болып табылады және Галиуллиннің белгілі классификациясына сәйкес қалпына келтіру міндетіне жатады.

Кездейсоқ бұзылулар тәуелсіз өсуі бар процестер класына жатады деп болжанады. Берілген мәселені шешу үшін ИТО стохастикалық саралау ережесі бойынша бұзылған қозғалыс теңдеуі жасалады. Бұдан әрі еругин әдісі квазиқайтару әдісімен біріктіріліп құрылады: 1) басқарушы вектор-функциялар жиыны; 2) Ито типінің екінші ретті берілген дифференциалдық теңдеудің берілген интегралдық көпбейне ие болуы үшін қажетті және жеткілікті жағдайларды қамтамасыз ететін диффузия матрицаларының жиынтығы.

Бұзу басқармасы бар стохастикалық мәселенің сызықтық жағдайы бөлек қарастырылады. Сызықтық қойылымда сызықтық емес рұқсат ету жағдайынан айырмашылығы, тәуелсіз өсуі бар процестер класынан кездейсоқ бұзылулар болған кезде, ұқсас сызықтық жағдайда, тәуелсіз винер процестері класынан кездейсоқ бұзылулар болған кезде рұқсат ету шарттарына сәйкес келеді. Сондай-ақ, бұзу басқармаларымен қалпына келтіру мәселесінің скалярлық жағдайы қарастырылады.

Түйін сөздер: ИТО-ның стохастикалық дифференциалдық теңдеуі, қалпына келтіру есебі, Мещерский есебі, интегралдық көпбейне, квазиқайтару әдісі.

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ОБ ОБРАТНОЙ СТОХАСТИЧЕСКОЙ ЗАДАЧЕ ВОССТАНОВЛЕНИЯ

Аннотация. В данной работе рассматривается общая задача восстановления в классе стохастических дифференциальных уравнений второго порядка типа Ито по заданным свойствам движения, когда управление входит в коэффициент сноса и методом квазиобращения определяется вид управляющих параметров, обеспечивающий необходимые и достаточные условия существования заданного интегрального многообразия. Далее приводится решение стохастической задачи Мещерского, которая является одной из обратных задач динамики и по известной классификации Галиуллина относится к задаче восстановления.

Предполагается, что случайные возмущения относятся к классу процессов с независимыми приращениями. Для решения поставленной задачи по правилу стохастического дифференцирования Ито составляется уравнение возмущенного движения. И далее методом Еругина в сочетании с методом квазиобращения строятся: 1) множество управляющих вектор-функций и 2) множество матриц диффузий, которые обеспечивают необходимые и достаточные условия того, чтобы заданное дифференциальное уравнение второго порядка типа Ито имело заданное интегральное многообразие.

Отдельно рассматривается линейный случай стохастической задачи с управлением по сносу**.** В линейной постановке в отличие от нелинейной условия разрешимости при наличии случайных возмущений из класса процессов с независимыми приращениями совпадают с условиями разрешимости в аналогичном линейном случае при наличии случайных возмущений из класса независимых винеровских процессов. Также рассмотрен скалярный случай задачи восстановления с управлениями по сносу.

Ключевые слова: стохастическое дифференциальное уравнение Ито, задача восстановления, задача Мещерского, интегральное многообразие, метод квазиобращения.

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